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Chapter 2
The mathematics of biologically effective EMFs: Are Maxwell's, Schrödinger's, and Pauli's formalisms compatible and complete?

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1 – Introduction

For the development of physical science (with biophysics being a segment thereof), the application of proper mathematical tools has had an enormous impact. In the theoretical elaboration of his concept of electromagnetism, Maxwell used Hamilton's quaternions which were a recent discovery in his days, but are replaced by the more convenient vector notation in contemporary physics. Schrödinger's famous equation is based on a complex-valued wave function (with the correspondence principle of classical physical magnitudes and imaginary quantum operators being no way arbitrary). Pauli, in turn, introduced the matrices named after him as constituents of a 2-component wave function in a squared complex space in order to represent 2-valued electron spin. At that point, the circle is closed, for it turns out that, apart from a unity matrix, Pauli's matrices are the exact match of a matrix representation of quaternions. So, we notice that all these formalism are indeed compatible, but their formal variety remains unsatisfactory.

The meaning of symbols denoting basic values, operators, wave functions etc., and the units corresponding to physical quantities, are compiled in **tables 1.1** and **1.2**.

The physical issue: How "real" is mathematics?

There lies a broader issue behind the topic of an appropriate choice of mathematical tools in order to represent physical conditions: What does mathematics tell us of "real" world? A typical question of this kind would be, if there is a "really existing" or "meaningful" equivalent to imaginary solutions of an equation. To give an example: the equations of electrodynamics yield, in the general case, a phase shift between alternating current and alternating voltage, giving rise to a complex-valued impedance ***Z*** as an extension of real-valued Ohmic Resistance *R*. At first sight, it might be supposed that only the real part of the current, i.e. the active current, has a physical meaning, and the imaginary part, i.e. the reactive current, might be omitted. But every electricity engineer would say that the reactive current is "real", too, in the sense of having physical and technological effects, e.g. unwanted energy losses through reactive power, and the induction of magnetic fields that is necessary for the action of transformers etc. So, it would be not reasonable to reject the imaginary part of AC at that point. Rather the calculated result (including imaginary currents) of the mathematical formalism demands attention, for it points to a part of physical reality (i.e. reactive currents) that is physically meaningful, though imaginary.

As David Hestenes demonstrated in details – a concise review of his work being given in [1, 2] –, that *geometric algebra* (GA) or multivectorial algebra offers itself as a unified mathematical language for physics, and in the guise of *spacetime algebra* (STA), explicitly includes relativistic physics. We will return to the power of STA in section 2.

The biological issue: How to retrieve fields that are relevant in biology?

From a *biological* point of view, the question may be expressed this way: Can mathematics help us to find out which kind of physical fields are biologically relevant? Biological and biophysical research is, to a large extent, confined to the action of classical force fields, such as electric field or magnetic field. However, when biological systems are recognized as quantum systems, we have to bear in mind that in this case potential fields take the place occupied by force fields in classical physics. So it seems reasonable to look for a wider selection of physical fields as causative factors for biophysical effects, with suitable mathematical tools as a guideline.

2 – Methods

Formalisms revised

In this sub-section, we will examine formalisms that were applied by Maxwell, Schrödinger, and Pauli as mathematical tools for tackling their respective problems. We will ask (i) for a possible unification of their formalisms (supposing they are compatible), and (ii) for their completeness, and finally look for (iii) a more comprehensive formalism that may indicate additional types of physical fields being of biological interest. In section 3, then, we will turn to experimental results that may be explained by a wider selection of bio-active fields.

Maxwellian electrodynamics

In his notation of scalar and vectorial magnitudes as *quaternions* with three "imaginary" or "vectorial" units **i**, **j** and **k**, James Clerk Maxwell anticipated the 4-vectors of relativistic electrodynamics such as energy – momentum, charge density – current density, and phase angle – angular momentum. Performing what appears, from a relativistic point of view, as a space-time-split, Maxwell applied the operator symbols *S* and *V* prefixed to products or derivatives of quaternions in order to denote that only the scalar, or vectorial part, resp., of the result was significant.

Here, the question rises: If quaternion representation was adequate to the physical nature of the problems in electrostatics and electrodynamics, would it not be an arbitrary truncation to omit either the vectorial or the scalar part of a solution? Is there a more comprehensive representation of electromagnetic theory that might include additional fields of biological interest?

Schrödinger's formalism of wave mechanics

When reading textbooks of quantum mechanics, the emergence of the *imaginary* unit i in Schrödinger's equation seems rather arbitrary, as does the *substitution of operators* such as $i\hbar\partial_t$ for energy, and $-i\hbar\nabla$ for momentum. But Schrödinger's choices were not at all arbitrary. Rather, the operators of wave mechanics as appearing in Schrödinger's equation for a free particle (1)

$$i\hbar\partial_t\Psi(\boldsymbol{x},t) = -\frac{\hbar^2\boldsymbol{\nabla}^2}{2m}\Psi(\boldsymbol{x},t) \tag{1}$$

(i.e. $i\hbar\partial_t$ and $-\hbar^2\nabla^2/2m$), when applied to a general 3-dimensional complex-valued wave function (2),

$$\Psi(\boldsymbol{x},t) = (2\pi)^{-3/2}\int\tilde{\Psi}_0(\boldsymbol{k})\,e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}\,d^3\boldsymbol{k} \tag{2}$$

reflect the dispersion relation $\hbar\omega = E = \boldsymbol{p}^2/2m = \hbar^2\boldsymbol{k}^2/2m$ [3].

For the imaginary factor i that remains somewhat mysterious in textbook physics, we will find a "real" interpretation concerning the spin.

Pauli's vector of matrices

In the Hamiltonian of a particle with charge q coupled to electromagnetic potentials $\boldsymbol{A}$ and Φ (3),

$$\hat{H}_{\mathrm{S}} = \frac{\hat{\boldsymbol{\pi}}^2}{2m} + q\Phi = \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A})^2}{2m} + q\Phi \tag{3}$$

a representation of spin was missed. Pauli's famous solution of this problem was to split the wave function into a 2-component spinor (4).

$$\Psi(\boldsymbol{x}, t) = \begin{pmatrix} \Psi_1(\boldsymbol{x}, t) \\ \Psi_2(\boldsymbol{x}, t) \end{pmatrix} \tag{4}$$

The Pauli-Schrödinger equation (5) of this spinor

$$i\hbar\partial_t\Psi = \left(\hat{H}_{\mathrm{S}} - \frac{\hbar q}{2m}(\boldsymbol{\sigma}\cdot\boldsymbol{B})\right)\Psi = \left(\frac{\hat{\boldsymbol{\pi}}^2}{2m} + q\Phi - \frac{\hbar q}{2m}(\boldsymbol{\sigma}\cdot\boldsymbol{B})\right)\Psi \tag{5}$$

introduces a vector $\boldsymbol{\sigma}$ of matrices σ_{k} (6):

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

Now, these matrices form an isomorphic representation of the basic vectors u_k of a 3-dimensional vector space within a 4-dimensional geometric algebra [3]. Via another isomorphism, this points to Maxwell's quaternions. Thus the circle is closed, and we conceive a consistent formal structure in Maxwellian electrodynamics and Pauli-Schrödinger quantum mechanics that finds its natural expression in terms of geometric algebra. *Maxwell's, Schrödinger's, and Pauli's formalisms are found to be compatible* in our revision.

Concerning the *completeness* of these formalisms (which is crucial for a comprehensive view on biologically relevant fields), we have to complete the (multivectorial) Grassmann basis in a way that it consists of (i) the three basic vectors u_k, (ii) three pseudovectors Iu_k, (iii) the scalar unit u_0 representing the real number 1, and (iv) the pseudoscalar I representing the imaginary unit i (i.e., in terms of geometry, a 90° rotation).

In the following section we will examine the physical meaning of the pseudoscalar I and thus illuminate the imaginary factor i appearing in the Schrödinger equation.

A real interpretation of quantum mechanics

As outlined in [1] and [3], application of geometric algebra to the Pauli-Schrödinger equation yields a *multivector representation* ψ *of a spinor* which follows the Pauli-Schrödinger-Hestenes (PSH) equation (7):

$$\partial_t \psi \, \hbar \boldsymbol{I\sigma}_3 = \hat{H}\psi + \frac{q}{2mc}(\boldsymbol{IB})\psi \, \hbar \boldsymbol{I\sigma}_3 \tag{7}$$

Here we find one of the basic vectors ($\boldsymbol{\sigma}_3$) arbitrarily chosen as a reference vector, and $\boldsymbol{IB}$ representing the magnetic field as a bivector. The term $\hbar \boldsymbol{I\sigma}_3$ may be interpreted as the double of *local spin bivector* $\boldsymbol{S}$ (8):

$$\boldsymbol{S} := \boldsymbol{Is} = \tfrac{1}{2}\hbar R \boldsymbol{I\sigma}_3 R^{\mathrm{t}} \tag{8}$$

with $\boldsymbol{s}$ being the local spin vector, and R the rotor component of the Hestenes multivector spinor (cf. 7).

So the $i\hbar$ in the familiar representation of the Schrödinger equation (cf. 1) is revealed as the double of spin bivector $\boldsymbol{S}$. This supports Hestenes' interpretation [1, 3] that the *Schrödinger equation* already includes spin and it *describes a particle in an eigenstate of spin*.

Including this interpretation, the PSH equation (7) represents a *complete* formalism (including spin) assigning a *real meaning to the imaginary multivectorial components*, i.e. the magnetic field bivector and the local spin bivector.

Completing the scope of bio-effective EMF

The result of the previous section shows the need for further inquiry in order to find a comprehensive formalism that will guide us towards a full coverage of electromagnetic fields (EMF) of biological importance. Special encouragement may be drawn from our intermediate results to apply geometric algebra in this endeavour. We have demonstrated the power of GA (i) to unify, (ii) to complete basic formalisms of physics, and (iii) to give imaginary components a real interpretation. This was explicitly demonstrated for quantum mechanics, but we also found that GA would provide an alternative to Maxwell's quaternion formalism that would be equivalent (because of an isomorphic basis), and even superior with respect to unification of fundamental physics (with the GA representation of Schrödinger's and Pauli's equations answering to the same formalism applied in electrodynamics). The potential that GA offers for unification within electromagnetic theory itself may be estimated from the amazing result that *Maxwell's four classical equations* are reduced to *one comprehensive, but simple equation* by introduction of the *multivectorial electromagnetic field* $F = \boldsymbol{E} + i\boldsymbol{B}$. That is why one can expect that the application of GA to EMF may also contribute to a focusing of biological and biophysical research on the types of fields that really matter. We will apply 4-dimensional GA with an underlying Minkowski metric which is termed space time algebra (STA).

STA applied to the electromagnetics of life

Liboff has argued [4] for an electromagnetic description for living beings expressed in terms of the *Hertz polarisation vector* Π. This may be regarded as a "hyperpotential" because the potentials of classical electromagnetic theory, namely the electrostatic potential Φ and the magnetic vector potential $\boldsymbol{A}$ (denoted as "electromagnetic momentum" by Maxwell), appear as derivatives of Π.

We represent this potential of higher order (9) in a classical manner, i.e. as a vector in 3-dimensional space with real-valued components α_k:

$$\Pi = u_1\alpha_1 + u_2\alpha_2 + u_3\alpha_3 \qquad (9)$$

From (9), the electromagnetic potentials and the electromagnetic force fields are derived through the spacetime split differential operator ∂ (10), e.g. in the first derivative:

$$\partial\Pi = \partial_t \Pi - \nabla . \Pi - i\nabla \times \Pi \qquad (10)$$

The results of derivation are compiled in **table 2.1**.

First of all, it should be stressed that although the Hertz vector, in agreement with classical electrophysics, is treated here as a 3-dimensional, real-valued vector, we apply the formalism of 4-dimensional spacetime algebra (including a 4-dimensional operator for the derivative) in order to demonstrate the result of derivation of Π with respect to spacetime coordinates. Therefore, all magnitudes appearing in the derivation are regarded as multivectors and denoted by italic letters rather than boldface letters for vectors. (An extended STA representation of the Hertz vector would be easily obtained by substituting complex-valued components for the real-valued α_k and adding a complex number as the scalar part. A full account of a comprehensive STA structure of electromagnetic hyperpotentials, potentials and fields remains to be published by the author [13].)

The *first derivative* consists of a scalar part which emerges as the divergence of Π , and two vectorial components. The first of these components is the time derivative of Π (denoted as Σ in table 2.1), and the second one (constituting the vector potential A) is the wedge product of ∇ and Π. (STA demands the pseudo-scalar $-i$ prefixed to the cross product in order to represent the wedge product.)

In the *second derivative*, we find the electric field E (consisting of the AC and the DC component) and the magnetic field B emerging as in Maxwell's classical formalism, but represented as a bivector due to its nature of a curl field. But additionally, there is a possible scalar component S that may be negligible under usual conditions. The equation gives a clue to experimental conditions that would produce a noticeable or measurable scalar force field component [5]: (i) a strong transient of electric potential (such as in nanosecond high voltage pulses), or (ii) a diverging current (such as the one which might occur in a spherical capacitor). Thus, the theory outlined here indicates types of conditions which might produce biological field effects that have rarely been subject to serious biophysical investigation so far.

Identification of possibly bio-active, non-classical fields

There is no doubt that *living organisms* are *electromagnetic systems* as much as they are the mechanical and biochemical ones. During last decades, a lot of theoretical and experimental evidence have been collected [6, 7, 8] for biological systems behaving as *quantum systems*, too. Consequently, living systems are sensitive to (i) electrical charges and currents, (ii) electromagnetic force fields, (ii) electromagnetic potentials and (iv) spin effects, the latter undoubtedly revealed by MRI (magnetic resonance imaging) which renders a detailed picture of biological activity in the human body.

For those electromagnetic bio-effects that are *not* attributable to classical force fields $\boldsymbol{E}$ and $\boldsymbol{B}$, causative factors need to be looked for among *potentials* (particularly the vector potential $\boldsymbol{A}$), *non-classical force fields* (scalar or longitudinal fields, of which the final word has not yet been said), and *potentials of higher order* such as the Hertz polarisation vector Π being superordinate to the potentials $\boldsymbol{A}$ and Φ.

The magnetic vector potential $\boldsymbol{A}$, or *electromagnetic momentum* according to Maxwell (indeed, this is a constituent of the "kinetic momentum" well known in quantum physics), is generally accepted as a physical reality, since the verification of the Aharonov-Bohm effect by Tonomura [9]. Its biological significance, though, has hardly been recognized so far. Having in mind that the potential $\boldsymbol{A}$ may play an essential role in quantum bio-effects (e.g. Josephson behaviour of cells, or coherence of brain partitions in conscious perception), one recognizes a serious need for research on immediate effects of electromagnetic potentials.

The existence of *scalar electromagnetic force fields* and their possible role in technology and biology has been only scarcely examined by serious theoretical and experimental methods. This is not only due to a certain reluctance of the scientific community, but rather to difficulties of evidencing measurable effects of suspected non-conventional electromagnetic force fields.

In the following sections we will give a brief review of experimental and therapeutic results from the application of non-classical fields (force fields and potentials, as well).

3 – Results

Experimental research

Results of experiments with "self-cancelling" coils

In the field of experimental electrobiology and quantum biology, different types of "self-cancelling coils" (with particular windings to cancel out – in total or in part – classical force fields) were applied to explore non-classical field effects [10]. In table 3.1, experiments of this type and their respective outcome are compiled.

Discussion

Resembling the setting of Aharonov-Bohm experiment, a coil wound up as a toroid will contain the magnetic field ***B*** in its interior, exposing exterior study objects to a field-free vector potential ***A***. When Mae-Wan Ho, in her studies of disruptive effects by weak magnetic fields on ontogenetic evolution, substituted a toroid for the solenoidal coil she had applied before, she found that the results were "quite tantalizing. Despite the fact that the magnetic field is negligible, significant increases in abnormalities (in pattern formation in *Drosophila* embryos) are found over matched controls, both when the embryos are in place before or after the power supply is switched on" [11].

Other types of coils that promote effects of non-classical fields are known als *Caduceus coils,* and *Möbius coils*. The latin term *Caduceus* denotes the staff carried by Hermes in Greek mythology. The staff is entwined by two serpents, a geometry that resembles the double helix of DNA. Coils wound in this Caduceus geometry do not cancel out the ***B*** field completely but shift the ratio of intensities *A*/*B* in favour of the ***A*** field. The impact on growth rates of human lymphocytes reported by Rein [10] is quite convincing for the biological significance of ***A*** being superior to the significance of ***B***. A *Möbius coil* is wound in the manner of the famous Möbius strip, so it forms a closed strip with only one side and a simple closed curve as its boundary. Möbius coils expose the biological model to non-classical fields and potentials in addition to the classical ***E*** field, producing results comparable to those of Caduceus coils.

The *bifilar* winding of solenoidal *coils* (with two parallel, but opposite windings) has a different characteristic: It cancels out rotatory ***A*** fields completely, as it would do for ***B*** fields and ***E*** fields. The significant effects thereof may be due to a curl-free part of the vector potential to which the door is open in classical electrodynamics. The results of plant treatment with water stimulated by a bifilar coil reported by Andocs *et al* [12] may be explained as an outcome of modification of water structure by the field of a bifilar coil. The latter was previously documented by Rein [10] as a significant shift in UV absorption at 244 nm that would not occur from treatment with a conventional solenoidal coil.

Therapeutic experience

Pulsed AC field therapeutic systems

Among the variety of non-thermal electromagnetic therapeutic treatments [16], pulsed electromagnetic field (PEMF) therapy is known to bring about remarkable healing effects, e.g. improved wound healing, quick reduction of edema and pain with acutely sprained ankles [14], or pain reduction with knee osteoarthritis [15]. As it is observed from the examples in Table 3.2, the characteristics of therapeutic devices in this field vary within a wide range.

Some devices apply weak magnetic fields in the order of magnitude of the geomagnetic field (such as Medicur), or of 1,000 times stronger (TheraCell), some apply extremely strong magnetic fields (PEMF 100). Common features of recent developments in the PEMF therapy are extremely low pulse frequencies, pulse durations in the range of microseconds, and *high voltage being built up and discharged within nanoseconds.*

A wide range of applications of high energetic impulse therapy (e.g. the TheraCell system) has been explored in *multicentre, double-blind, placebo-controlled, randomized studies* in human as well as in veterinary medicine [17-22]. Among those are (i) pain therapy (lasting relief of chronic pain from bones, joints etc.), (ii) accelerated healing (following bone fractures, rheumatism, arthrosis etc.), (iii) accelerated regeneration of muscular traumata, and further indications for (iv) neurological diseases, and (v) urological and gynaecological diseases.

Discussion

Manufacturers argue that the magnetic induction would play a crucial role for therapeutic success by tissue penetration down to approx. 20 cm or more. But the threefold twisted treatment loop of the TheraCell system exhibits a winding that effectively *reduces* magnetic fields about conductors. Of course, the biological outcome may be partly assigned to ELF frequencies of pulses (as pulse modulations of a RF carrier wave that convey a bio-active signal).

The author's attention was drawn to the TheraCell device where a therapeutist reported extraordinary treatment success after few single treatment sessions. What is special about systems of this type? Let us assume that an electric potential of 30 kV was discharged within 3 ns. This results in a voltage transient of 10^{13} V/s. The equivalent scalar field strength (cf. section 2) seems to be sufficient to provoke unusual effects within cells that are obviously involved in the healing process. (Definitely it is not any thermal effect of the energy conveyed that acts curative here.) In analogy to the electric field strength in the order of magnitude of 10^7 V/m across the cell membrane, that prompts – according to Froehlich's reasoning – long-range (spatial) coherence in biological systems, the enormous voltage transient produced e.g. by the TheraCell system could restore *temporal coherence* resulting in a "reset" of the cell.

4 – Conclusions

The formalisms of Maxwell, Schrödinger, and Pauli, though different, lay a *compatible* basis for attributing biological effects to physical quantities. Integration and, at the same time, simplification of different formalisms is achieved by application of multivectorial algebra that is appropriate to *complete* the formal basis. The first and second derivatives of the Hertz polarisation vector with respect to spacetime coordinates comprise the potentials and force fields well-known from Maxwellian theory as well as possible scalar force fields.

Serious research points to a *significant biological effect of non-classical fields, particularly potentials*, which is supported by the perception of quantum properties of living beings.

According to multivectorial theory of electromagnetism, *extreme voltage transients,* as applied in the newest type of *pulsed electromagnetic field therapy* devices, give rise to scalar force fields accountable for extraordinary therapeutic effects.

From both theoretical as well as experimental and therapeutic point of view, there is an urgent need for refined and practical means in the *measurement of non-classical fields*.

Research in biological and medical application of vector potentials and scalar fields should be promoted.

Table 1.1. Meanings of quantum physical symbols

Symbol	Physical meaning
Ψ	Schrödinger's wave function
$\hat{H}_S$	Schrödinger's Hamiltonian (energy operator)
$\boldsymbol{\sigma}$	Pauli vector of unit matrices σ_k
ψ	Multivector representation of a spinor (Pauli-Schrödinger-Hestenes equation)
R	Rotor component of Hestenes multivector spinor
I	Pseudoscalar (rotational operator corre-sponding to imaginary unit i)
$\boldsymbol{s}$	Local spin vector
$\boldsymbol{S} = I\boldsymbol{s}$	Local spin bivector

Table 1.2. Meanings of physical symbols, and corresponding units. It should be noted that the bivector (i.e. imaginary) part of derivatives (cf. section 2 and table 2.1) contains the velocity of light c as a factor. It is a convention in spacetime algebra (STA) to set the value of c as 1, but for the resulting physical quantities, the dimension has to be corrected by a factor of $1/c$. Therefore, the corresponding units involve an additional factor of $(m/s)^{-1}$. E.g. the unit of the electroscalar potential Φ as the divergence (real derivative) of the Hertz polarization vector Π is V (Volt), whereas the unit of the magnetic vector potential A as the curl (imaginary, or bivector derivative) of Π is T.m (tesla times meter) which equals V.s/m = $V.(m/s)^{-1}$. Accordingly, the tesla itself, as the unit of magnetic induction B, equals $V.s/m^2 = (V/m).(m/s)^{-1}$, corresponding to V/m as the unit of electric field strength E.

Symbol	Physical meaning	Unit
$\boldsymbol{x}$	Position vector	m (meter)
t	Time	s (second)
$\boldsymbol{k}$	Wave vector	m^{-1}

ω	Angular velocity	s^{-1}
m	Mass	kg (kilogram)
E	Energy	J (Joule)
$\boldsymbol{p}$	Momentum	kg.m/s
h	Planck constant	J.s
$\hbar = h/2\pi$	Reduced Planck constant	J.s
J	Electric current	A (Ampère)
q	Electric charge	A.s
$\boldsymbol{B}$	Magnetic induction	T (Tesla) = $V.s/m^2$
Φ	Electric scalar potential ("electro-static" potential)	V (Volt)
$\boldsymbol{E}$	Electric field strength	V/m
$\boldsymbol{A}$	Magnetic vector potential (Maxwell's electromagnetic momentum)	T.m = V.s/m
$\boldsymbol{\pi} = \boldsymbol{p} - q\boldsymbol{A}$	Kinetic momentum	kg.m/s
Π	Hertz polarisation vector	V.m
Σ	Vector potential of scalar field S	V.m/s
Ξ	Temporal derivative of Σ	$V.m/s^2$
S	Scalar field strength	V/s

Table 2.1. Spacetime algebra (STA) derivation of electromagnetic potentials and fields from a 3dimensional Hertz polarisation vector Π. In the third row (*2nd derivative*), S denotes a scalar field strength which is derived from a curl-free part of a vector potential Σ (constituted by the temporal variation of Π). In fact, S consists of two equivalent terms. Their physical interpretations are (i) the temporal variation of electric scalar potential Φ and (ii) the divergence of currents (represented by $\nabla . \Sigma$). Both physical quantities have the unit of V/s. The curl of Σ and the equivalent temporal variation of A yield the AC electric field, and the gradient of Φ the DC electric field.

hyperpotential	$\Pi = \quad u_1\alpha_1 + u_2\alpha_2 + u_3\alpha_3$
1st derivative	$\partial\,\Pi = -\ \nabla.\Pi\ +\ \partial_t\Pi\ \ - i\nabla\times\Pi$ $= \quad \Phi \quad + \quad \Sigma \quad + \quad A$
2nd derivative	$\partial^2\,\Pi = \partial_t\Sigma - 2\nabla.\Sigma - 2i\nabla\times\Sigma - \nabla\Phi - i\nabla\times A$ $= \ \Xi + S + \qquad E \qquad + \ B$

Table 3.1. Results of experiments involving "self-cancelling coils" to exhibit effects of non-classical electromagnetic fields and potentials

Author (year)	**Ref.**	**Type of coil**	**Type of field(s)**	**Experimental model**	**Outcome**
Ho *et al* (1994)	[11]	Toroid	***A*** (***B*** negligible)	Pattern formation in *Drosophila* embryos	Increasing (!) abnormalities found over matched control (B=0.5-9 mT)
Rein (1990)	[10]	Caduceus	***A, B***	Growth rate of human lymphocytes treated with square wave form (peaking around 4 kHz)	Acceleration 3.5 times stronger than with solenoidal coil
Rein (1989)	[10]	Möbius	***E,*** *?*	Growth rate of human lymphocytes treated with LF ***E***-field peaking around 260 Hz	1.8 fold acceleration (p = 0.001, n = 7)
Rein (1988)	[10]	Möbius	***E,*** *?*	Inhibition of neurotransmitter uptake of PC12 nerve cells	1.2 fold enhancement (p = 0.05, n = 6)
Rein (1996)	[10]	Bifilar	Curl-free vector potential?	UV (224 nm) absorption of treated water (43 kHz sine wave, negligible ***A*** and ***B*** reported)	>5 fold increase compared to solenoidal coil with equal power output (p = 0.001, n = 6)
Andocs *et al* (2009)	[12]	Bifilar	Curl-free vector potential	Harvest properties of beta-red carrott seeds treated with coil-stimulated water	Increasing harvest mass and water content of harvest, decreasing dry-content of harvest

Table 3.2. Comparison of three types of pulsed electromagnetic field (PEMF) therapy devices

Name of device	***Medicur***	***PEMF 100***	***TheraCell***
Type of carrier field	Magnetic field from soft iron core	50 Hz em. field	RF em. field (200…230 kHz)
Pulse frequencies	3 Hz; 7.8 Hz; 20 Hz	up to 50 Hz	1 Hz…3 Hz
Pulse duration		4 µs	100…150 µs
High voltage		13 kV	15…30 kV
Rise and decay time	1 µs (rise) 10 µs (decay)		nano-seconds
Magnetic induction	< 50 µT	up to 1.92 T	40…80 mT
Tissue penetration	30 cm	not specified	18 cm

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